

# DYNAMIC MULTIDIMENSIONAL SCALING

*ANDRZEJ SOKOŁOWSKI*

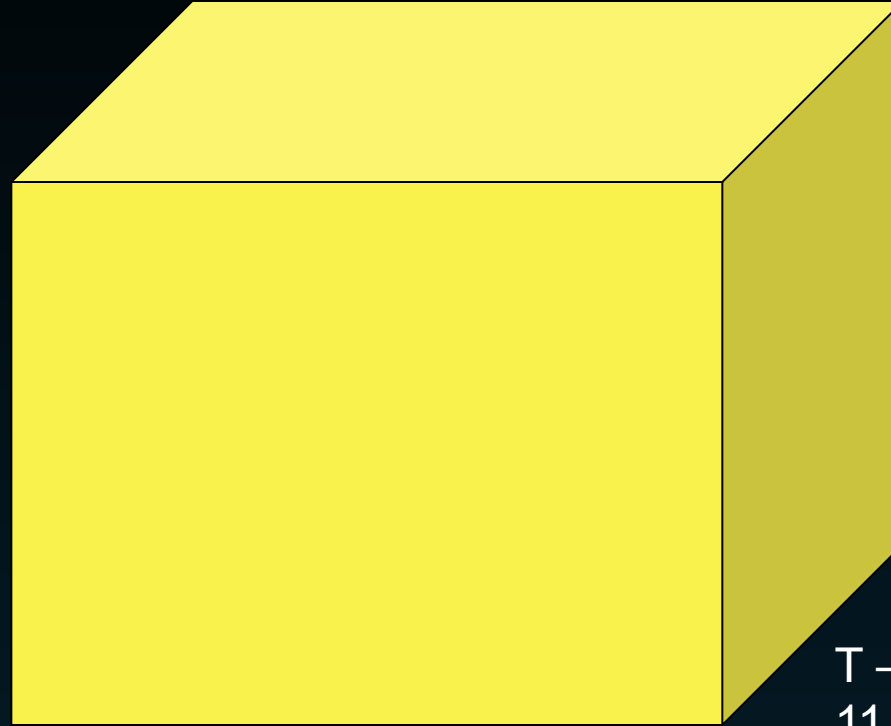
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*WROCLAW UNIVERSITY OF ECONOMICS AND BUSINESS*

# Data Cube

Y - objects  
27 EU countries



Z – variables  
5 variables

T – Time units  
11 Years: 2014-2024

*Multivariate Spatio-Temporal Data*

# Multidimensional Scaling (MDS)

Multidimensional scaling (MDS) is a popular and well established procedure in multivariate analysis. It allows for optimal graphical representation of multidimensional objects configuration in low-dimensional space, usually on the two-dimensional plane. Distance matrix in original space is an input data for the procedure which is looking for such configuration of objects which produce the distance matrix in output space minimizing similarity measure for original and output distances (such as stress - [7]). Details can be found in textbook on MDS, like [1, 2, 3, 4, 5, 8, 9].

Walesiak M., Dehnel G., Dudek A., Visualisation of linear ordering results using multidimensional scaling – problems and an overview of studies, *Argumenta Oeconomica*, 2025, Vol. 54, No 1, 187-203

# International Conference on Information Complexity and Statistical Modeling in High Dimensions with Applications (IC-SMHD-2016)

*May 18-21, 2016, Nevsehir, Turkey*

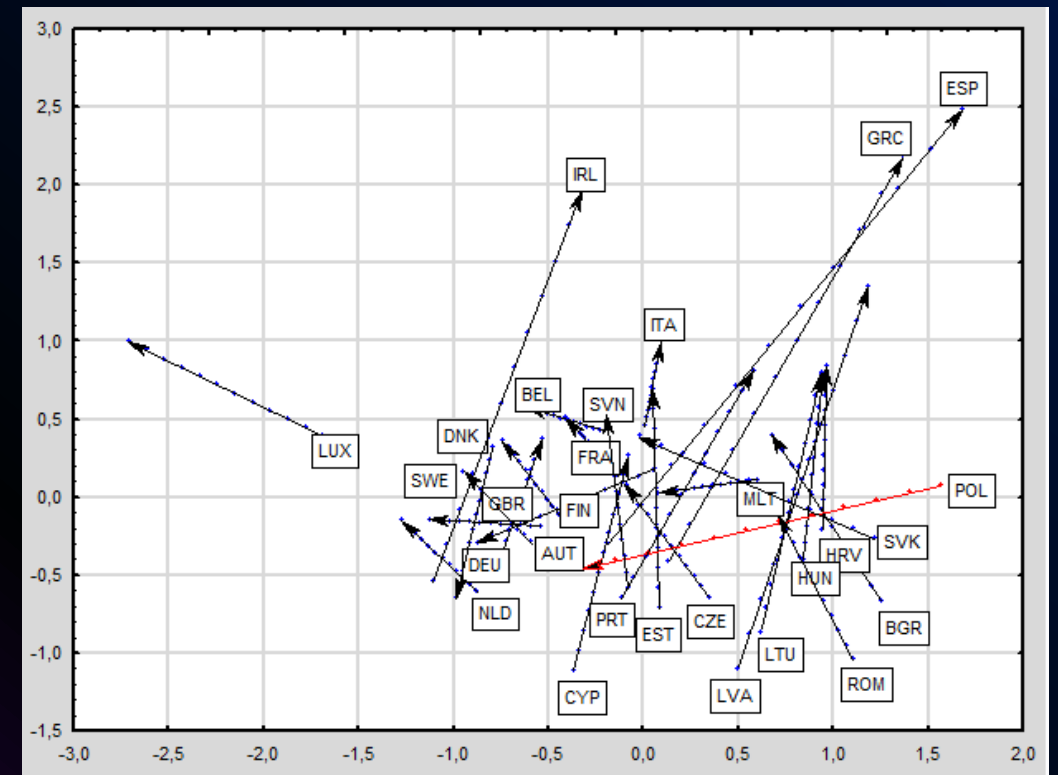
## **Predictive Multidimensional Scaling. The Analysis of EU and Turkey Economic Development**

Andrzej Sokołowski and Małgorzata Markowska and Agnieszka Rygiel



# APPROACH 1

- **Global standardization** – using mean and standard deviation, calculated for the whole period
- MDS **separately** for each year
- **Linear** trends for two coordinates
- Clustering trends



Conference of  
Classification and Data  
Analysis Section of the  
Polish Statistical  
Society (SKAD)

*October 23-25, 2017,  
Kraków, Poland*

# NIELINIOWE DYNAMICZNE SKALOWANIE WIELOWYMIAROWE

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## **APROKSYMACJA WYNIKÓW DYNAMICZNEGO SKALOWANIA WIELOWYMIAROWEGO TRENDAMI NIELINIOWYMI**

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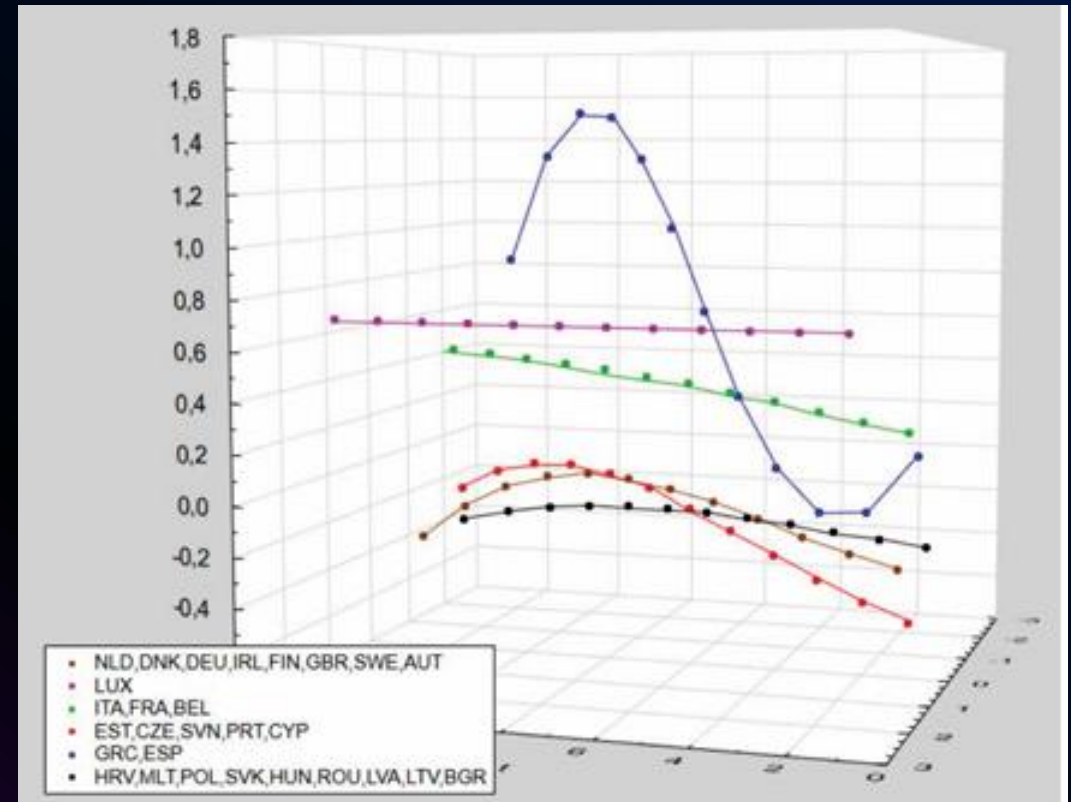
## **NONLINEAR TREND APPROXIMATION OF THE RESULTS OF DYNAMIC MULTIDIMENSIONAL SCALING**

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DOI: 10.15611/pn.2018.507.16  
JEL Classification: C14, C23, O11

## APPROACH 2

- **Global standardization** – using mean and standard deviation, calculated for the whole period
- MDS **separately** for each year
- **Non-linear** trends for two coordinates
- Clustering trends



## APPROACH 3 (NEW)

- **Local standardization** – using mean and standard deviation, calculated for each period
- MDS **separately** for each year
- **Chain rotation**
- Non-linear and linear trends for two coordinates

$$x_1^* = x_1 \cos \alpha - x_2 \sin \alpha$$

$$x_2^* = x_1 \sin \alpha + x_2 \cos \alpha$$

$$D = \sum_{i=1}^n d(y_{i,t-1}; y_{i,t})$$

# VARIABLES

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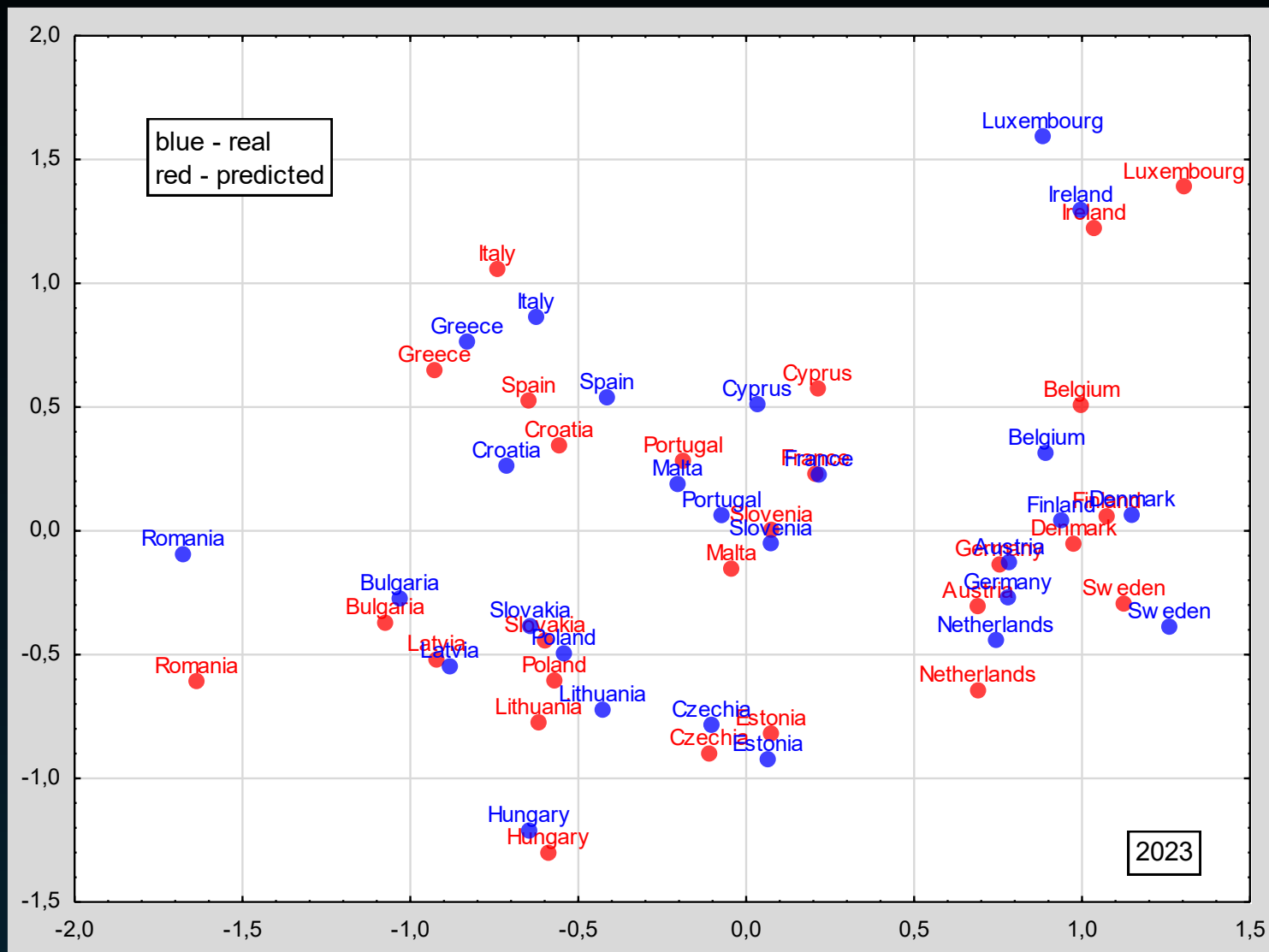
- Gross domestic expenditure on R & D (GERD) – percentage of GDP, all sectors
- Gross domestic product (GDP) at current market prices – Purchasing power standard (PPS), per inhabitant
- Total employment – from 20 to 64 years: percentage of total population
- SII – Summary Innovation Index (average of 32 indicators)
- Harmonised Index of Consumer Prices (HICP) – annual average, all-items HICP

# ROTATIONS

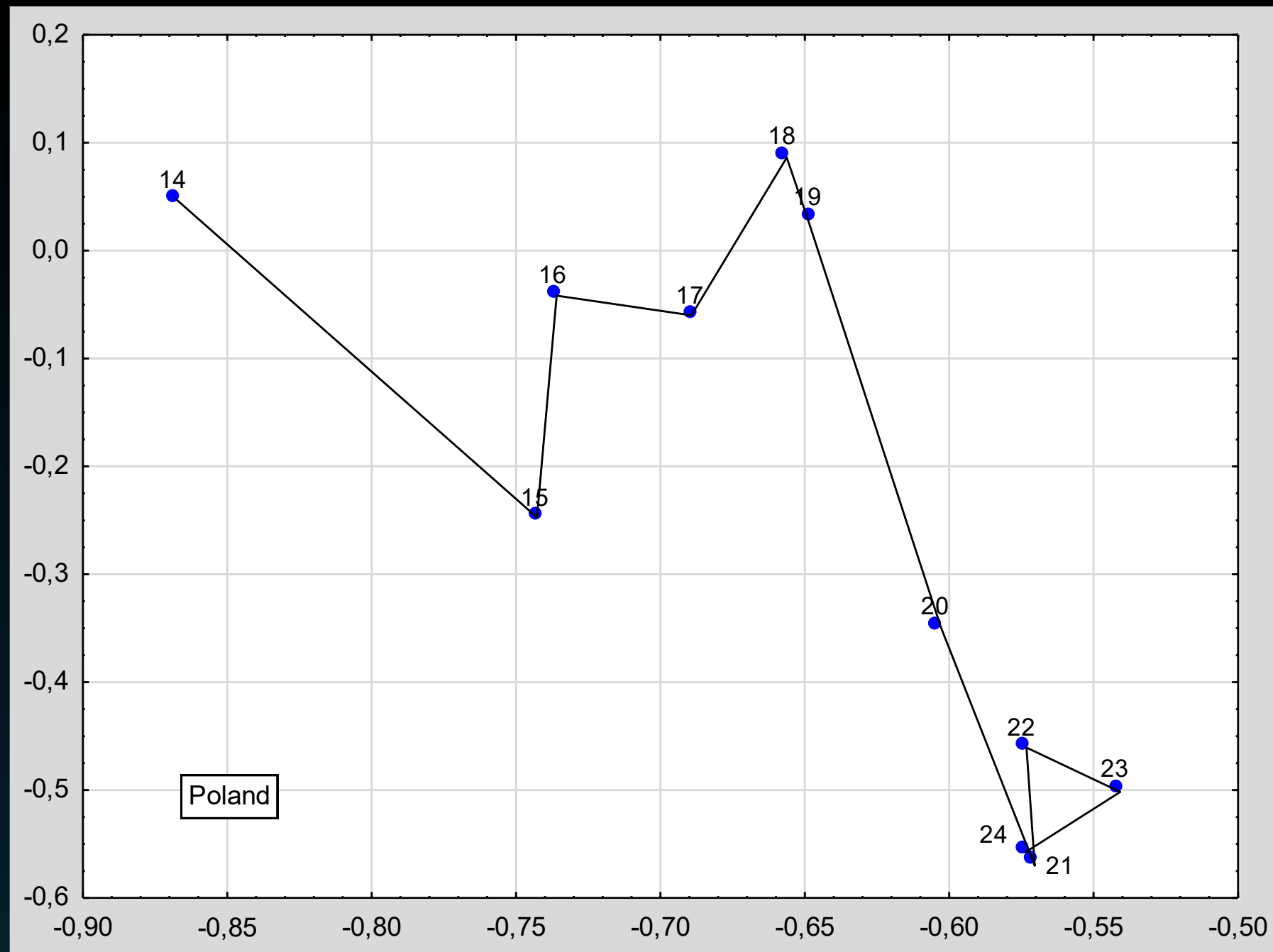
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| Year               | 2015  | 2016  | 2017  | 2018  | 2019  | 2020  | 2021  | 2022  | 2023  | 2024  |
|--------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Angle<br>(degrees) | 3     | -6    | -3    | 2     | 8     | 12    | 12    | 24    | 25    | 27    |
| $d(C_t, C_{t-1})$  | 3,380 | 2,566 | 1,158 | 0,780 | 0,773 | 0,867 | 0,579 | 1,357 | 0,523 | 0,393 |

# Prediction vs Actual - for 2023 - Approach 3

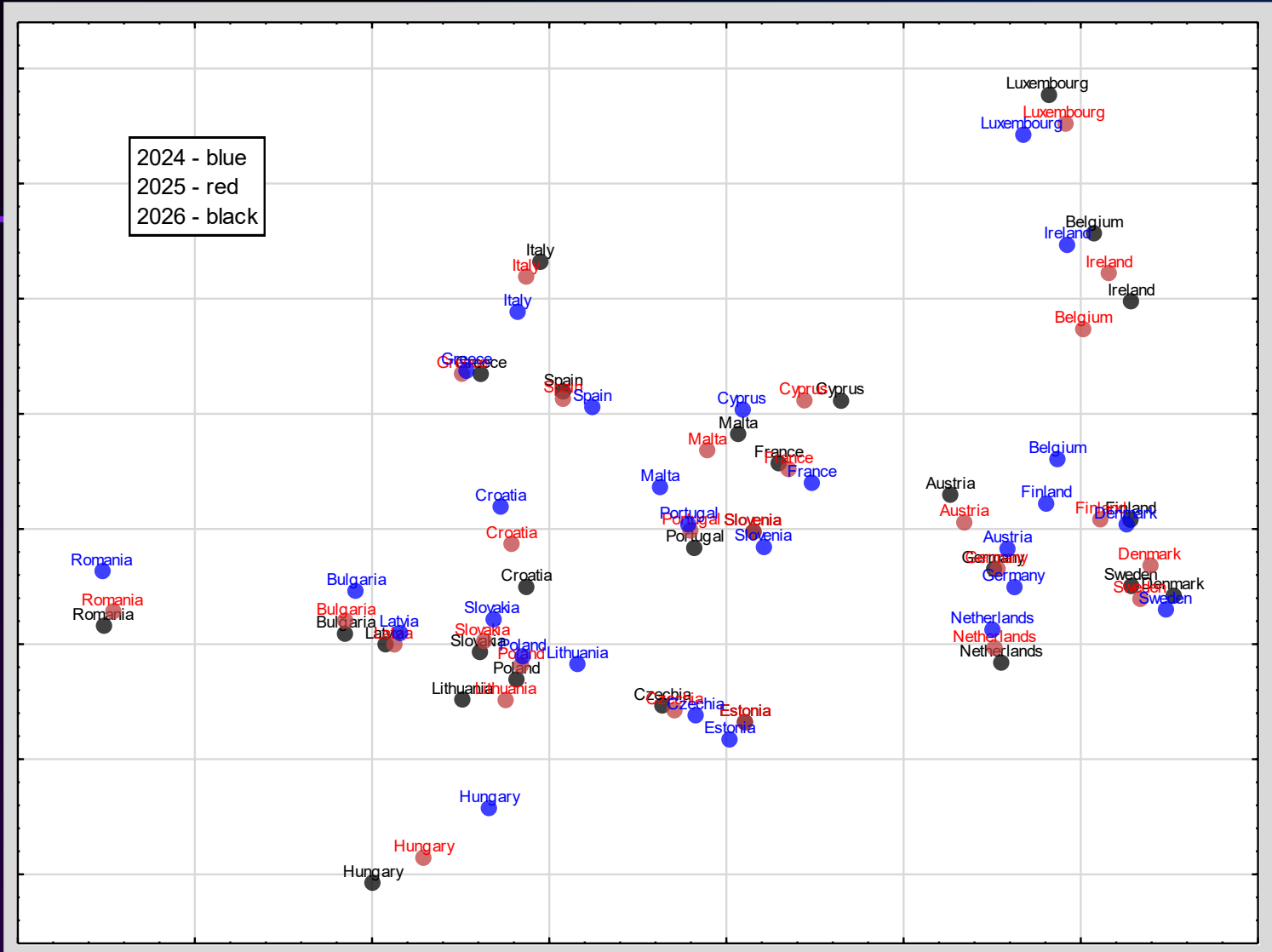


Average  
error: 0.046



# FORECASTS APPROACH 3

| Year     | 2024  | 2025  | 2026  |
|----------|-------|-------|-------|
| Angle    | 27    | -1    | -1    |
| Distance | 0,393 | 0,924 | 0,637 |

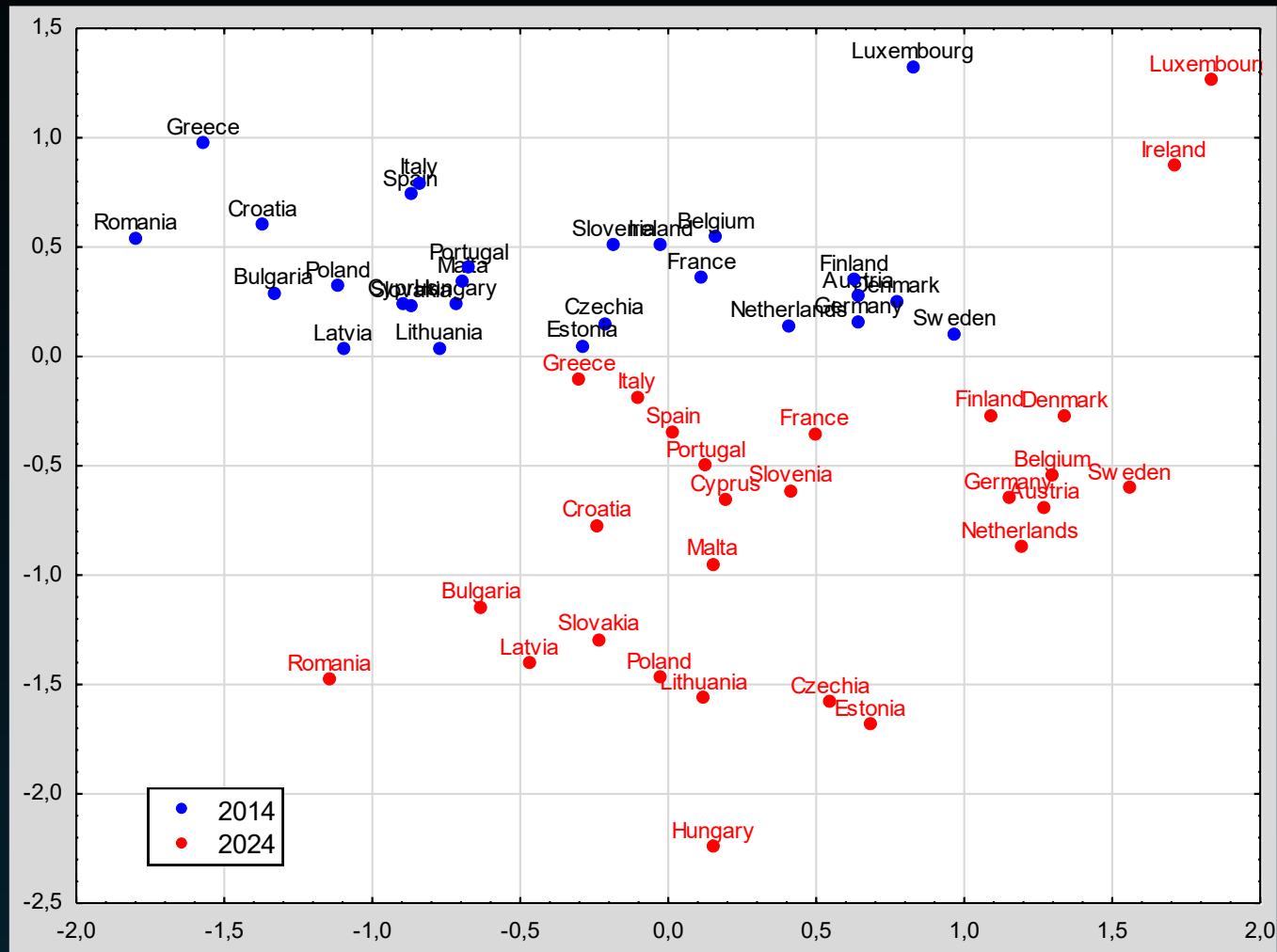


## APPROACH 4 (NEW)

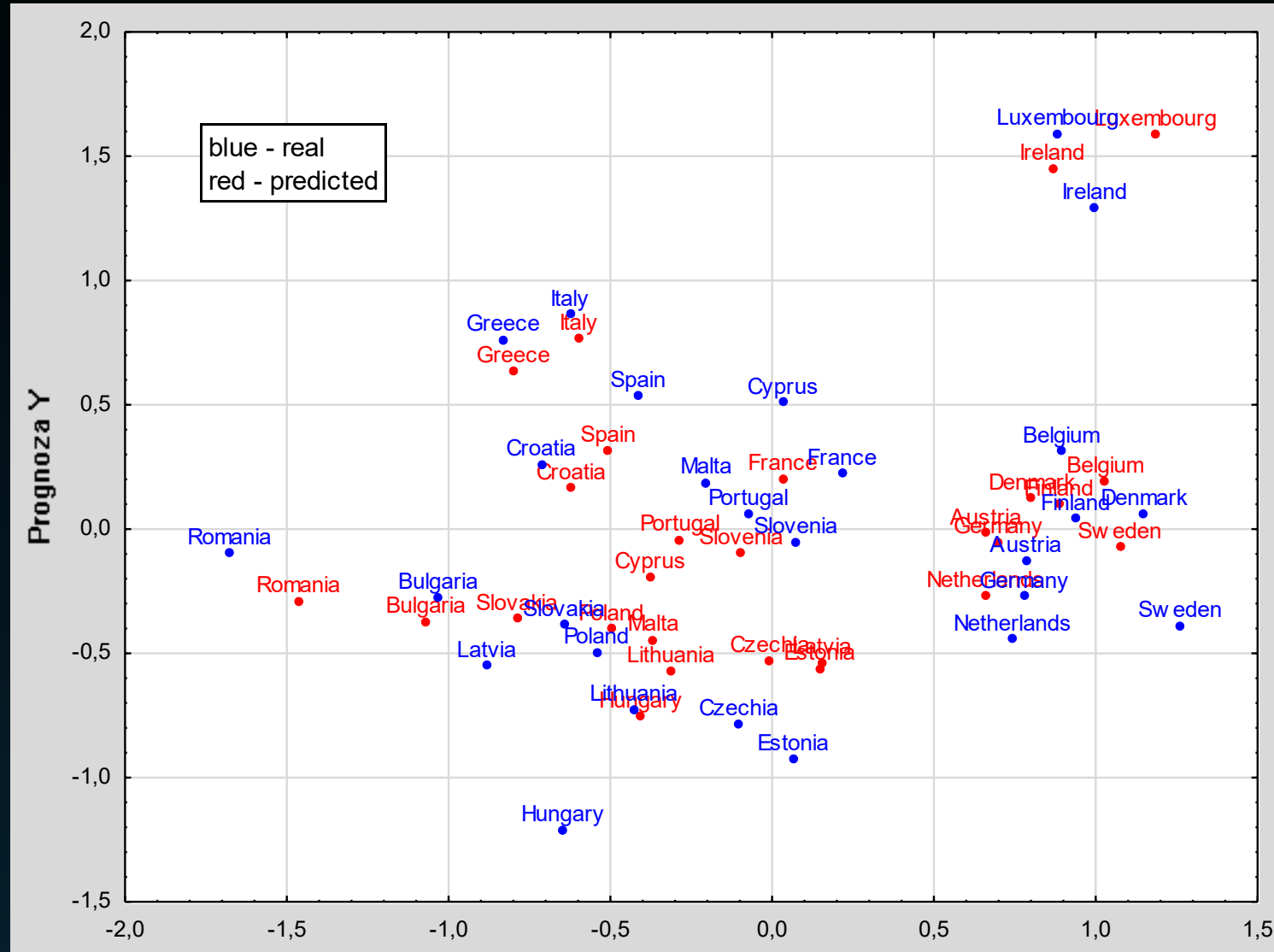
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- **Global standardization** – using mean and standard deviation, calculated for the whole period
- MDS **globally** for the whole period
- **Non-linear** trends for two coordinates
- Standardization for each year

# Approach 4



# Prediction vs Actual - for 2023 – Approach 4



Average  
error: 0.133

# SUMMARY

| Approach                  | 1                | 2                | 3           | 4                     |
|---------------------------|------------------|------------------|-------------|-----------------------|
| Standardization           | global           | global           | local       | global                |
| MDS                       | local            | local            | local       | global                |
| Transformation            | none             | none             | rotation    | none                  |
| Trends                    | linear           | non-linear       | BE          | BE                    |
| Transformation of results | trend clustering | trend clustering | none        | local standardization |
| Advantage                 |                  |                  | forecasting | movement              |

**THANK YOU  
FOR  
YOUR  
ATTENTION**