Główny Urząd Statystyczny

V Kongres Statystyki Polskiej

A Novel Density-Based Metric for Measuring Spatial Competition in Urban Agglomerations

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Where we are – and what we still need?

Existing tool-kit

Classical spatial-competition models Hotelling (1929); Salop (1979) → now implemented in agent-based computational economics (Tesfatsion & Judd 2006).

Industry & financial-market concentration metrics (Soldatos 2021)

- Structural: Herfindahl-Hirschman Index (HHI), C5/C10 ratios (McAuliffe 2015)
- Behavioural: price–cost-margin measures such as the Lerner Index (Lerner 1934; Shaffer & Spierdijk 2017)
- Hybrid: indicators blending market-share and cost-margin information.

Spatial agglomeration statistics Ellison & Glaeser (1997); Duranton & Overman (2005); Kopczewska (2019, 2023).

Research gap

We need a single, synthetic measure of spatial market concentration integrating structural, behavioural, and spatial dimensions.

Spatially-Weighted HHI: bridging structure & space

Why it matters

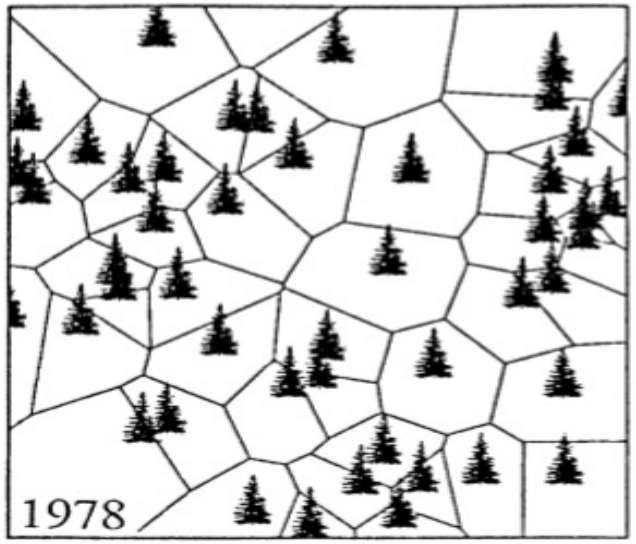
- Marries a classical concentration metric (HHI) with Voronoi catchment areas and population weights, capturing where customers really are
- Provides a single number that integrates structural and spatial dimensions of competition.
- Next step: add behavioural depth (price–cost margins) to obtain a fully hybrid index.

Index	Market Share	Price	Tied to physical location	Catchment Area
HHI	Yes	No	No	No
C10	Yes	No	No	No
Lerner	Yes	Yes	No	No
Proposed Index: Spatially Weighted HHI	Yes	(pending)	Yes	Yes

Call for feedback

We will present this new index at the forthcoming workshop and **welcome critical comments and suggestions for refinement.**

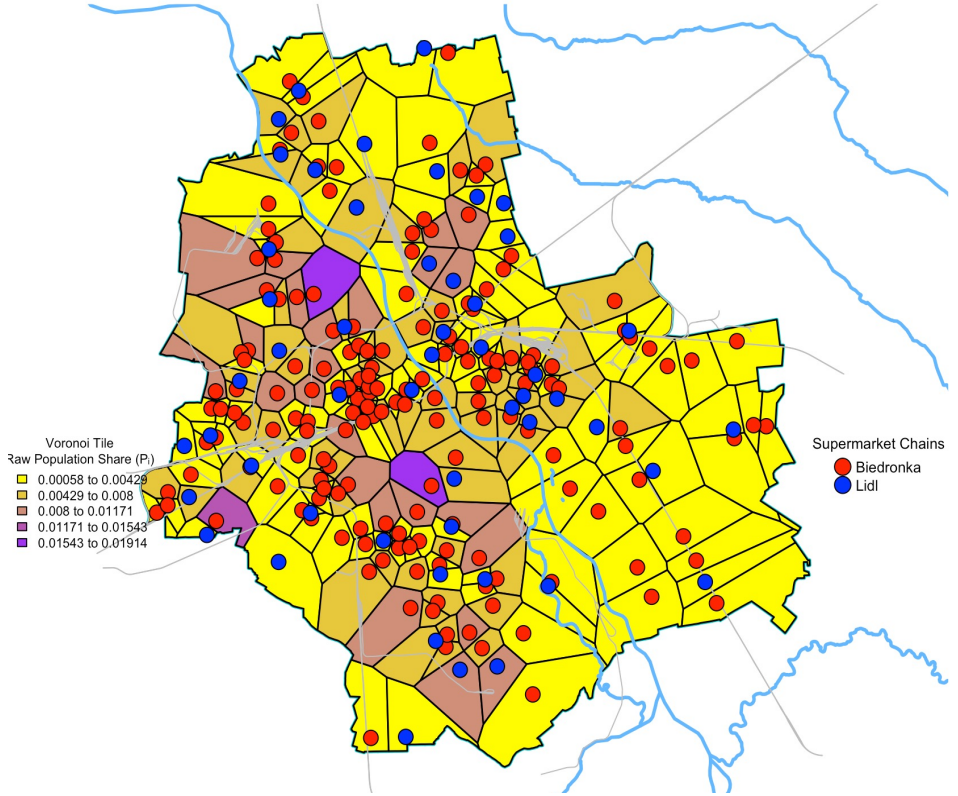
Inspiration — Linking Ecological Competition Models with the Herfindahl-Hirschman Index (HHI)



Spatial Competition Models for Plant Populations (Kenkel 1991)

+ Herfindahl-Hirschman Index

→ Spatially Weighted HHI



Voronoi Tesselation:

a) all individuals appear simultaneously, and remain fixed in position;

b) all individuals are equally 'weighted';

c) tiles 'grow' at the same rate in all directions;

d) the growth rates of tiles are linear, and the same for all individuals;

e) tile growth ceases when neighbours (polygon edges) contact.

Economic analogue: the biological resource corresponds to market demand (population) distributed across the landscape.

Use of Spatially Weighted Market Shares and HHI

- **Location Points** serve as seeds for creating Voronoi polygons V_i. **Population Assignment:**
- Population proportion P_{ij} assigned to each Voronoi polygon from CENSUS population grid C:
- $P_{ij} = P_j \cdot \frac{Area(C_j \cap V_i)}{Area(C_j)}$
- where P_j is the population of grid cell C_j.
- The total population of each Voronoi polygon P_i is thus:
- $P_i = \sum_j P_{ij}$

- **Spatially Weighted Market Shares,** Population density p_i within polygon V_i, where A_i is the area of Voronoi polygon :
- $p_i = \frac{P_i}{A_i}$
- Spatially weighted market share S_i for economic units' location:
- $S_i' = p_i \cdot w_i$
- $S_i = \frac{S_i'}{\sum_k S_k'}$
- Normalize shares ensuring they sum to 1.

- Aggregated from location-level shares S_i to economic units' location shares S_c.
- Market concentration adjusted by spatial population density:
- $HHI_{spatial} = \sum_c S_c^2$ where $S_c = \sum_{i \in c} S_i$
- Spatially weighted HHI provides insights into market dominance, adjusted for spatial distribution and population density.

Behaviour of the Spatially-Weighted HHI under alternative location-and-population scenarios

Experiment design (2, 4 and 8 outlets)

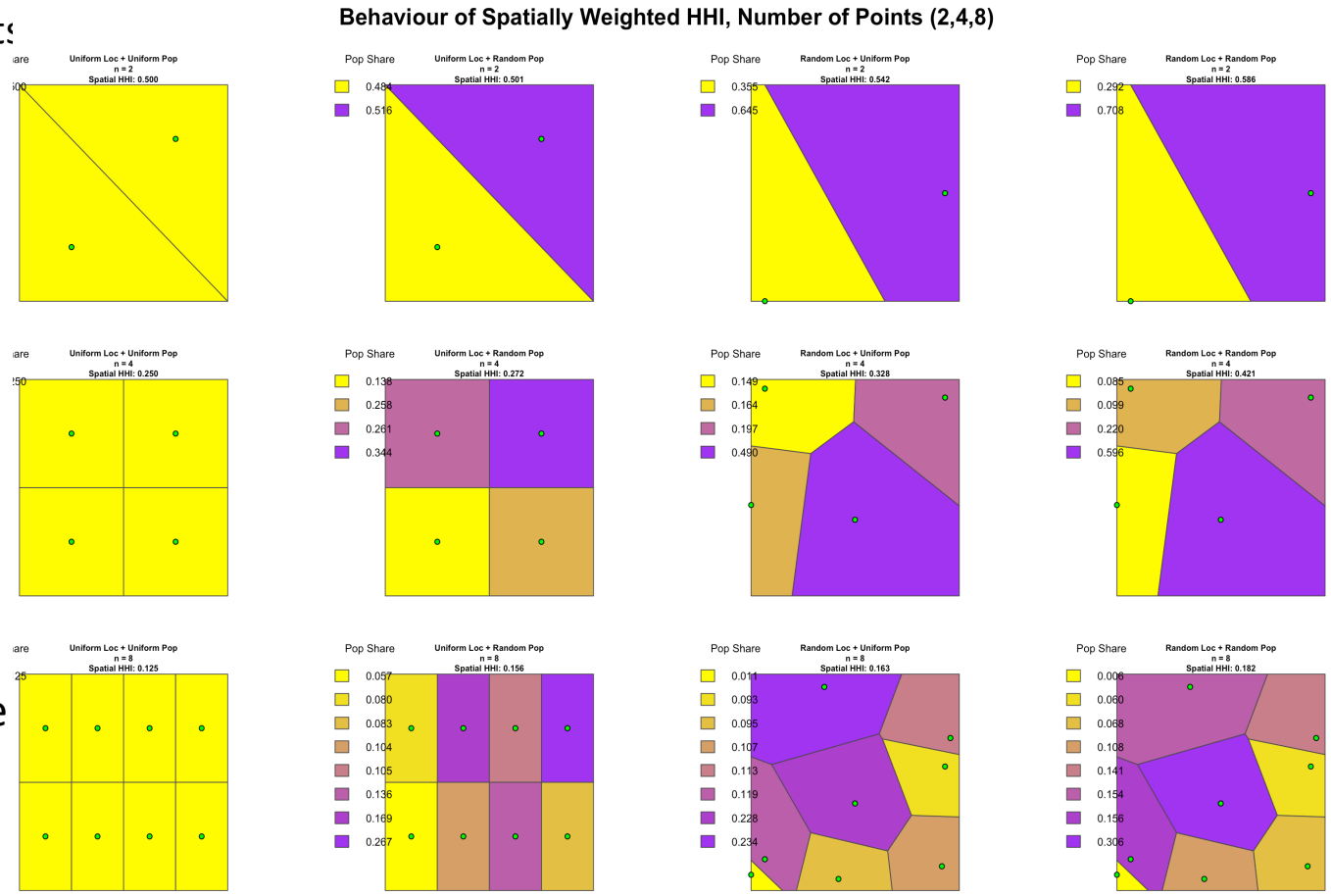
- Regular locations + Uniform population – reference case, lowest concentration.
- Regular locations + Random population – demand pockets lift concentration.
- Random locations + Uniform population – irregular catchments raise concentration.
- Random locations + Random population – both forces combine, highest concentration.

Key Findings:

Index rises whenever either outlet placement or demand density becomes more uneven.

Extreme equality (case 1) yields the theoretical minimum; combining randomness in both layers (case 4) pushes the index toward its maximum.

Confirms that the Spatially-Weighted HHI responds intuitively to spatial and demographic heterogeneity, validating its diagnostic power.



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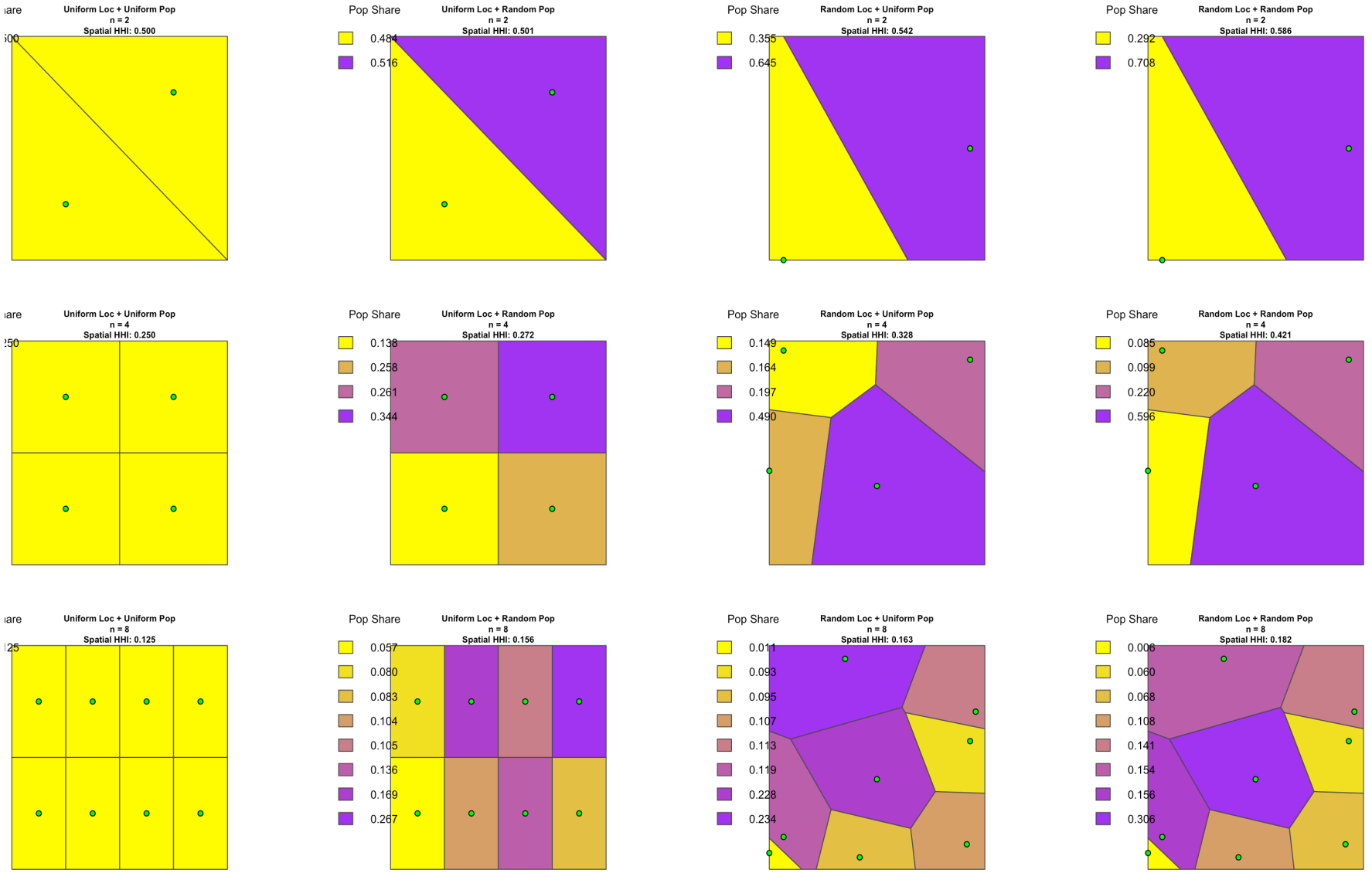


Two Case Studies Using the Spatially-Weighted HHI

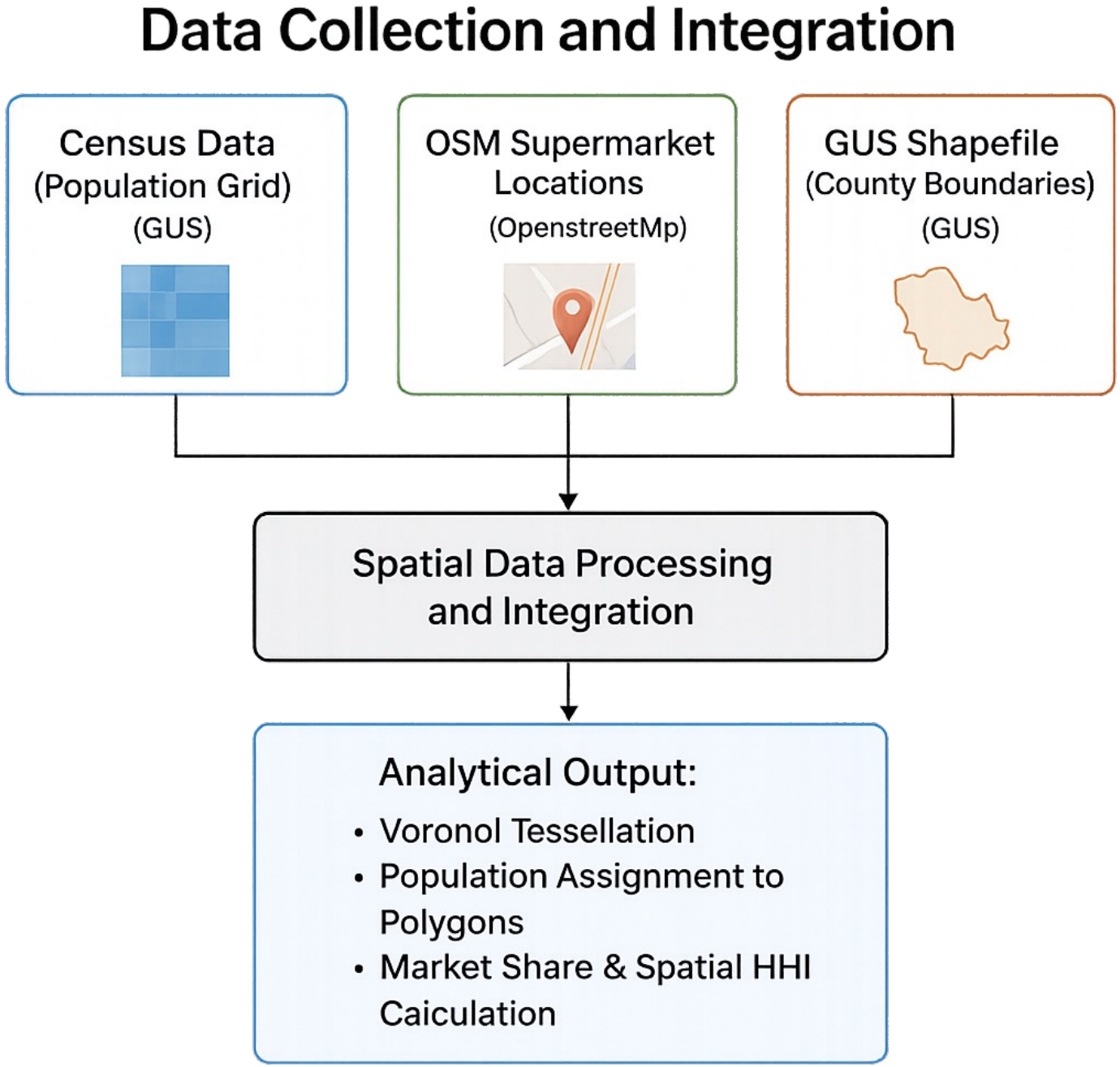
Supermarket rivalry: Biedronka vs Lidl, Warsaw

- Regulatory analytics – spatial-competition assessment for antitrust oversight
- Network strategy – optimal positioning of new outlets from each chain's perspective

Competition for pupils: primary schools, Warsaw

- Urban-planning tool – identifying locations for new schools to relieve overcrowding.

- **Population Census Data (GUS):**
  - Polish National Census GRID NSP 2021.
  - Provides detailed spatial distribution of population at fine-grained census grid level.
- **Locations from OpenStreetMap:**
  - Supermarket chains (e.g., Lidl, Biedronka).
  - Precise geographic coordinates, polygons, and points for retail locations.
- **County Shapefiles (GUS):**
  - Official administrative boundaries (powiaty).
  - Essential for constraining spatial analyses and visualizations.



Supermarkets – What the Spatial-Weighted HHI Reveals and How We Use It

1 | Spatial concentration metrics

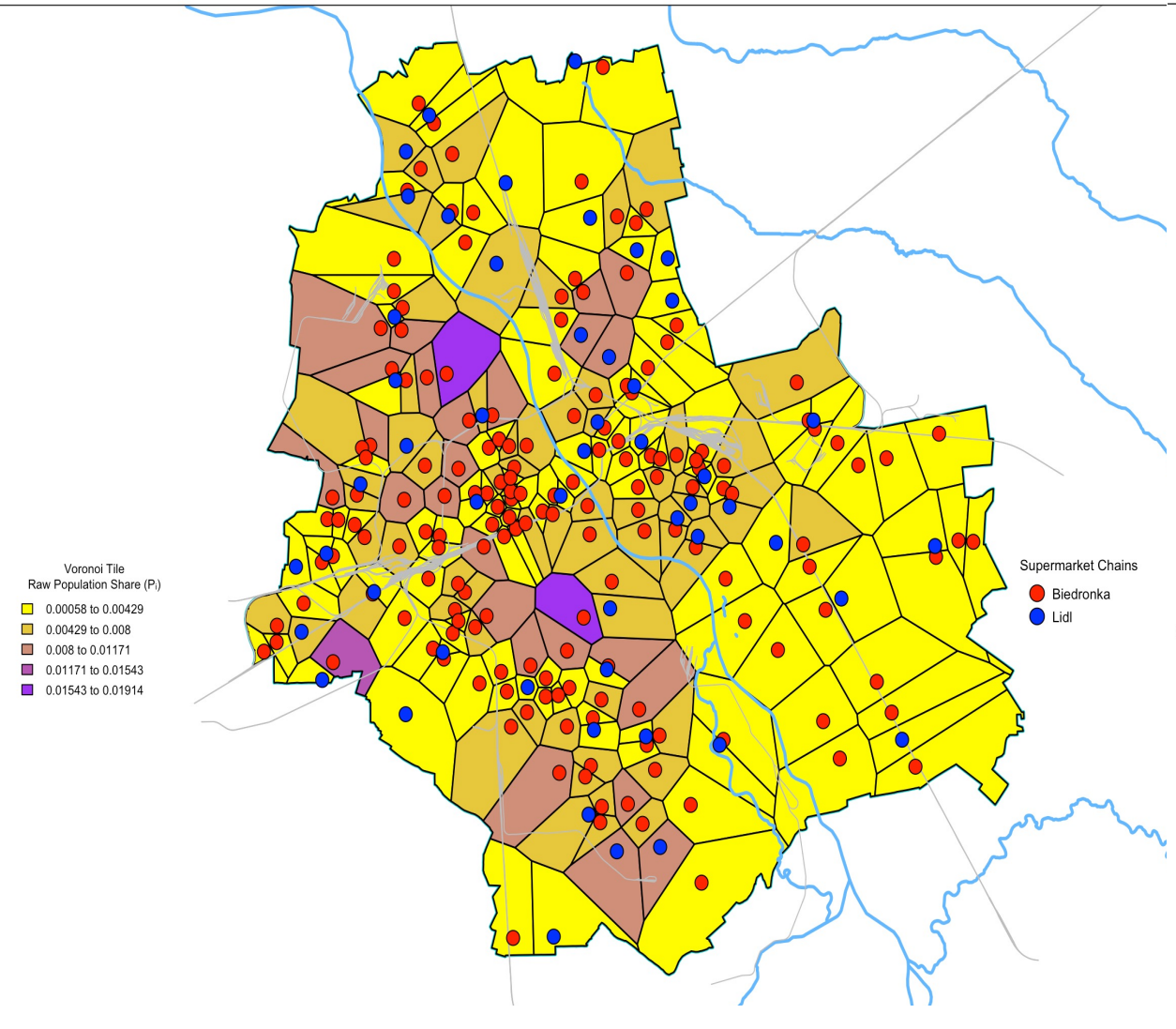
- Computed a Spatial HHI for each chain by overlaying Voronoi catchments and weighting them with population density.
- Bench-marked against a simple population-only HHI to show the extra insight gained from adding geography.

2 | Why the spatial version adds value

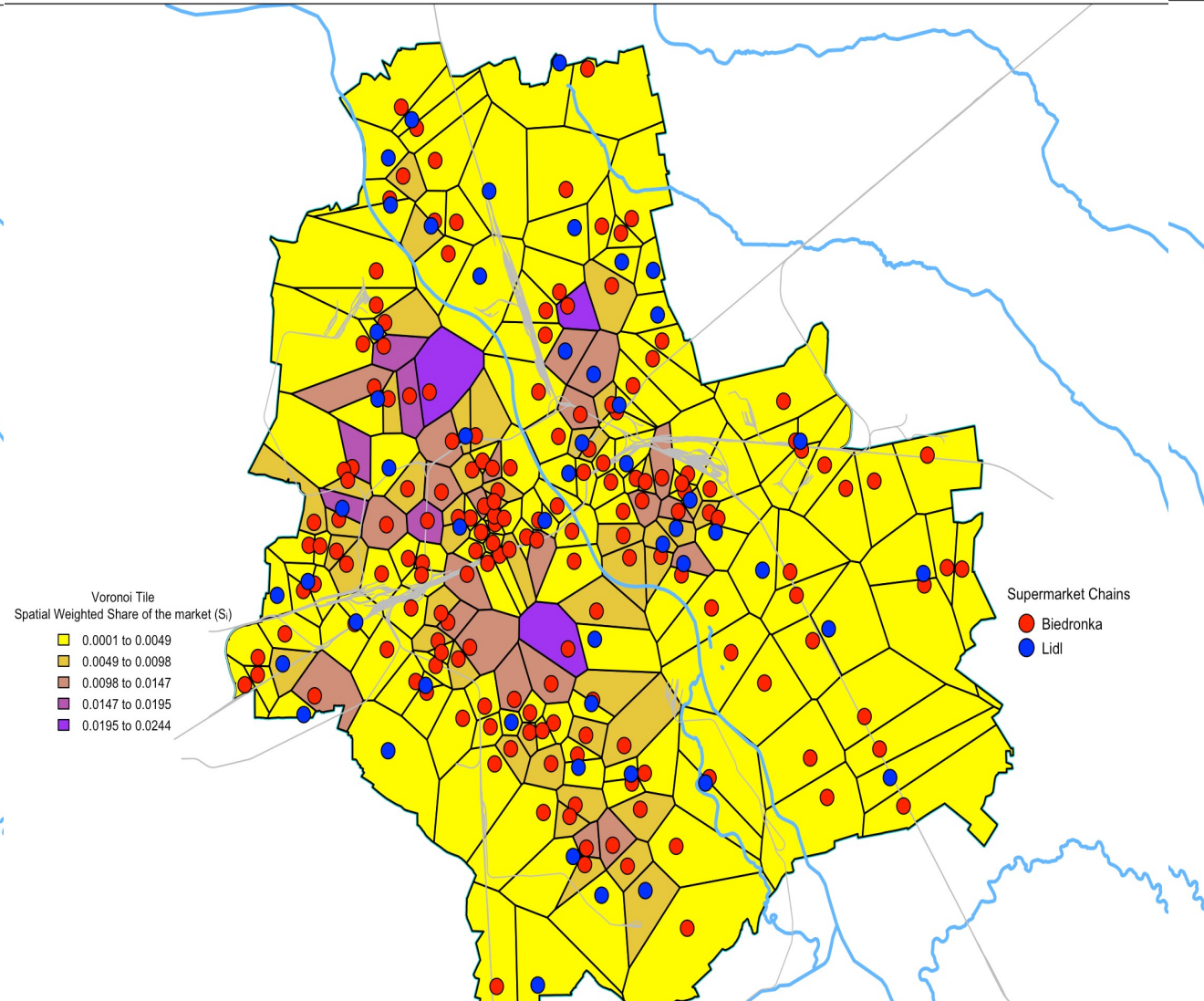
- Makes visible the local “pockets” where one chain dominates despite a city-wide duopoly.
- Flags small high-demand areas that a classic, aspatial HHI would treat as perfectly competitive.

Comparing Density-Weighted and Population-Based Spatial HHI

Supermarket Market Share (Raw Population Only)



Supermarket Market Share by Population with Spatial weightening based on density



Supermarkets – What the Spatial-Weighted HHI Reveals and How We Use It

3 | Optimising the new store location

Ecology analogy: a new outlet competes both intra-species (with its own chain) and inter-species (with rivals). Our search algorithm scores candidate sites on:

- Distance from the chain's existing stores – limits cannibalisation.
- Resident population inside the prospective Voronoi tile – maximises demand.
- Distance to current Voronoi borders – secures room for a large catchment.

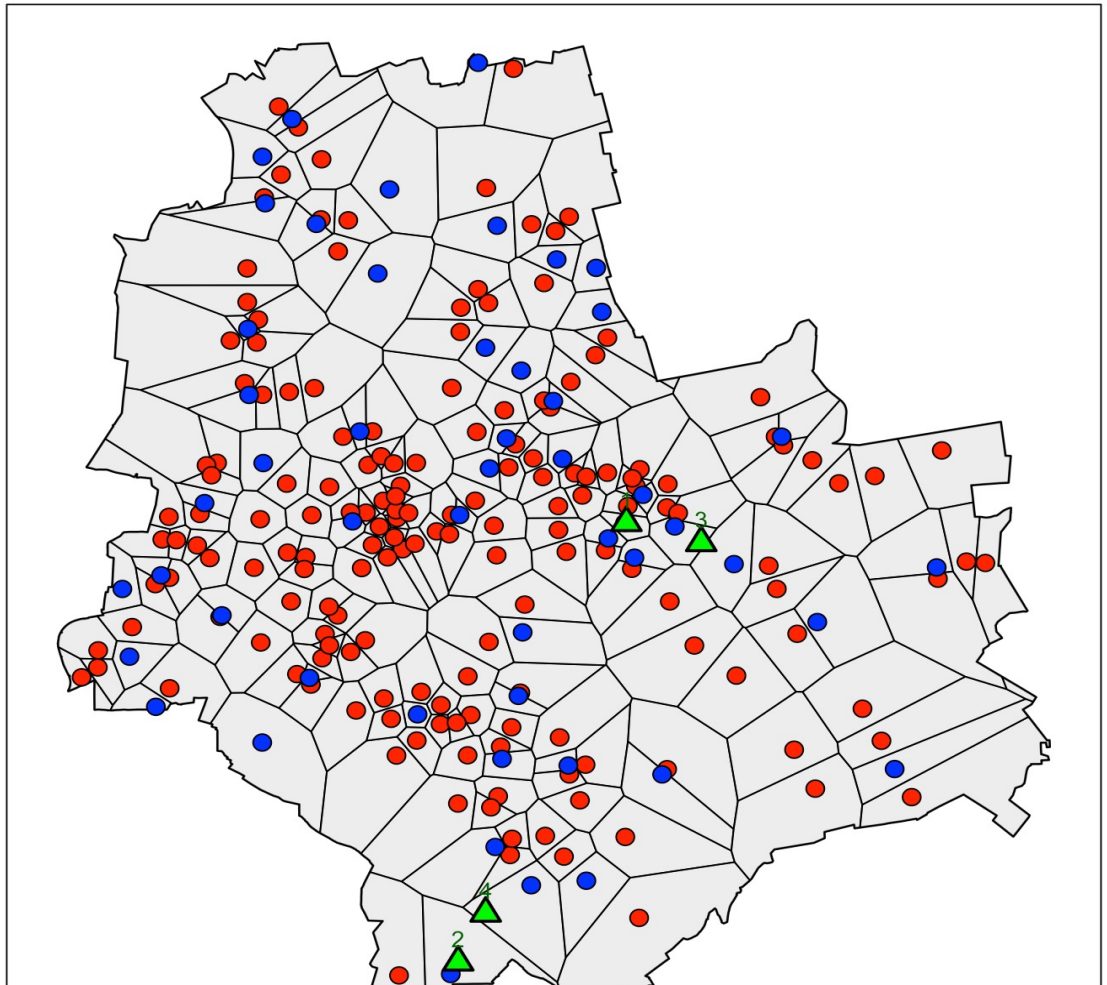
4 | Decision support

- The algorithm outputs a shortlist of high-leverage sites that lower the chain's Spatial HHI (healthier competition) while raising its effective market reach.
- Regulatory bodies can use the same metric to assess whether proposed openings reduce or amplify spatial concentration.

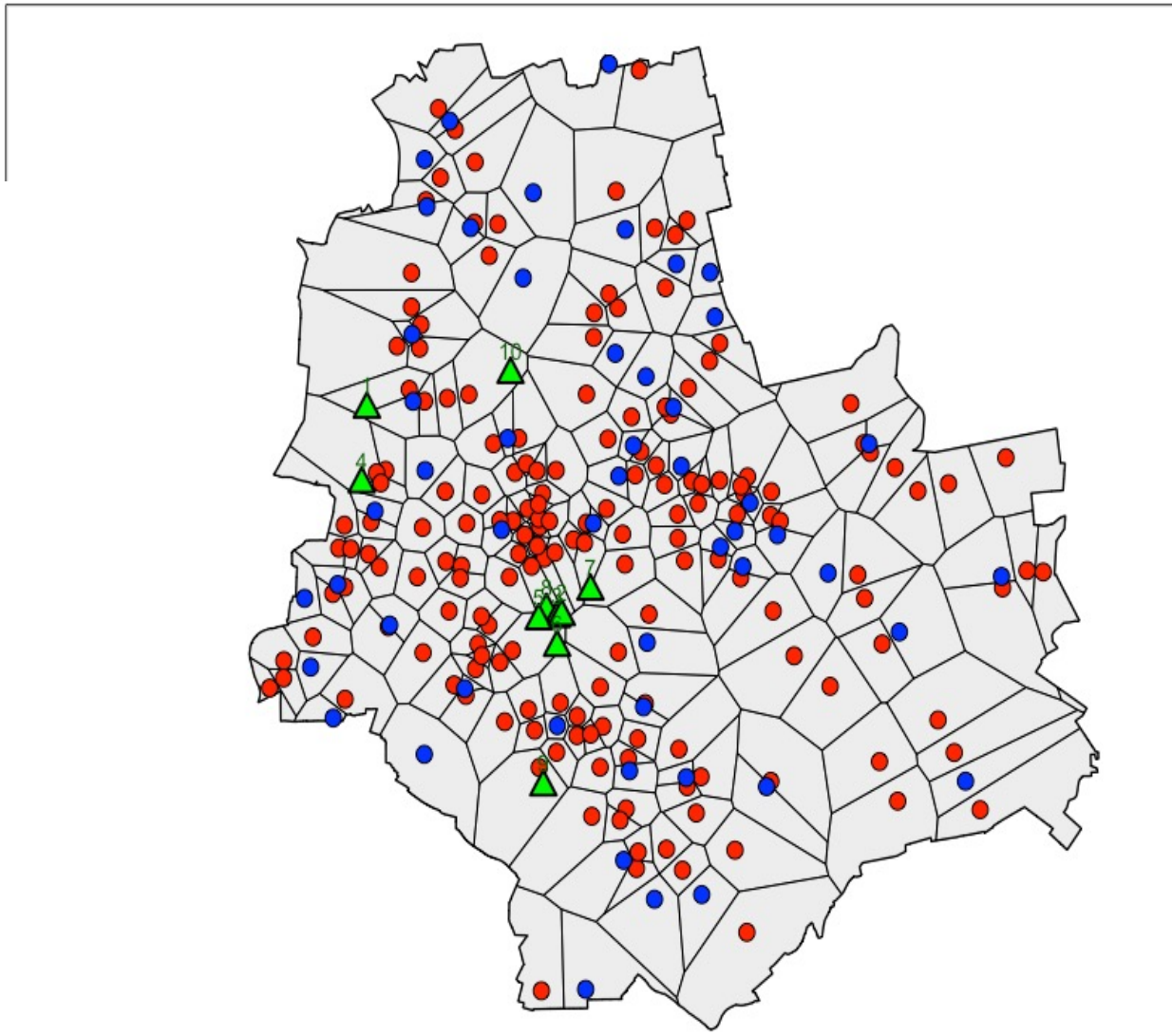
Supermarkets New Locations

Algorithm determining the location relies on distance do the same
shop, increase of population and distance to the border of Voronoi

Biedronka **Top Candidate Locations**



Lidl **Top Candidate Locations**



Primary-School Catchments, Warsaw – Using the Spatial HHI to Tackle Overcrowding

Current diagnosis Spatially weighted HHI = 0.018 ⇒ peripheral schools carry a disproportionate share of the city's 0-14 population.

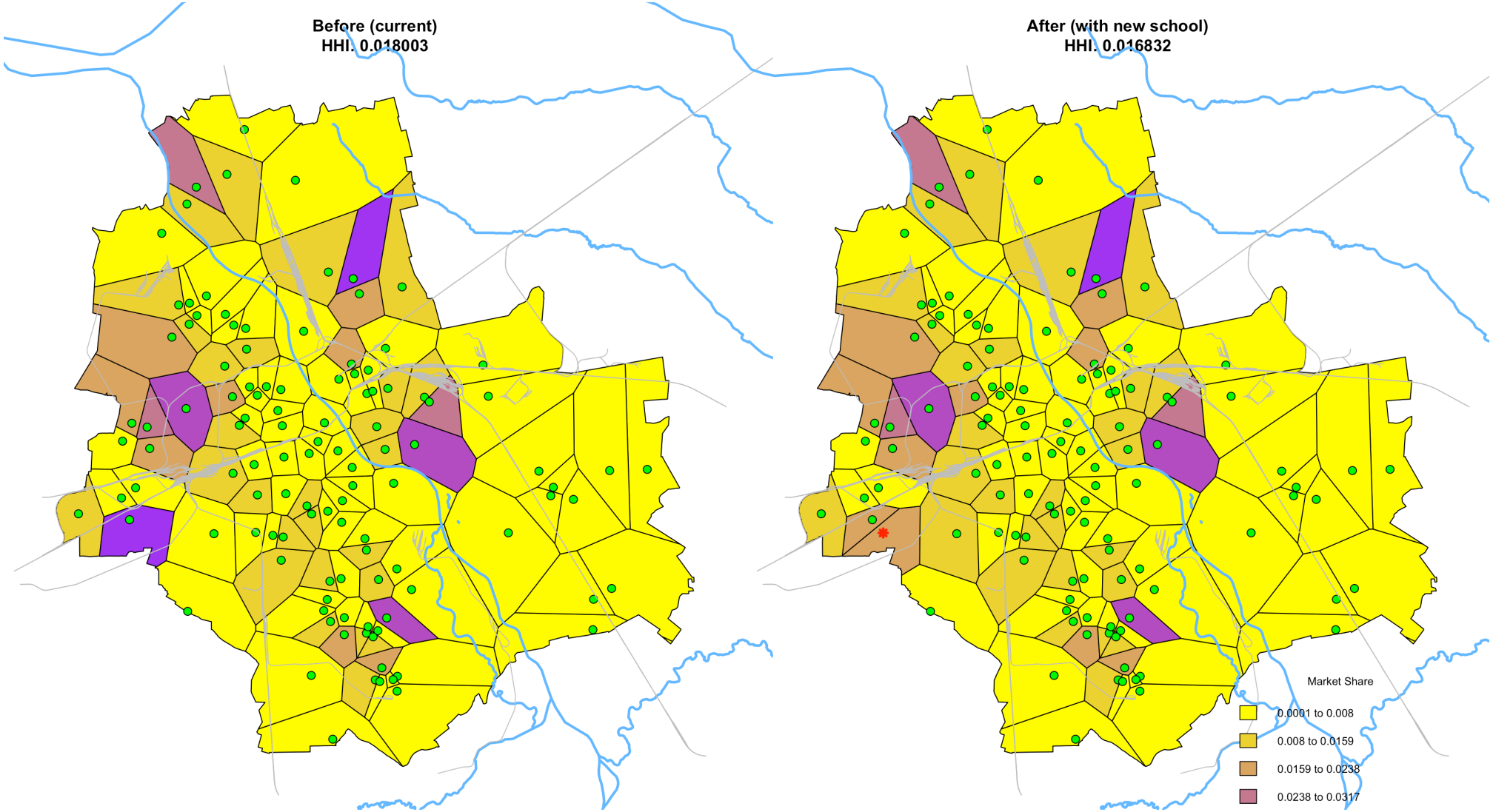
Goal Pinpoint new school sites that minimize the city-wide spatial HHI, thereby evening out pupil loads.

Search algorithm

- Generate feasible candidate plots (zoning + land-use filters).
- Recompute Voronoi catchments and age-specific population for each candidate.
- Choose the site(s) that yield the largest drop in spatial HHI.

Pilot result Adding one school at the suggested location lowers the index to 0.016 and relieves the peripheral hot-spots (maps: before vs after).

Competition for pupils: primary schools, Warsaw



(Green dots = schools; darker tiles = higher catchment-weighted share of pupils.)

Next Steps

Incorporate physical barriers Factor in rivers, rail corridors and large green belts when delineating catchments.

Build a fully hybrid index Extend the Spatial HHI with price / cost-margin data for retail chains.

Stress-test spatial scale Re-compute at alternative statistical levels (NUTS-0, NUTS-2, LAU-2) to check robustness.

Network-level competition Create an indicator that treats an entire chain (multi-site network) as a single competitive unit.

Domain-specific weighting schemes Define bespoke spatial weights for other services—e.g. hospital emergency rooms, fire—where demand drivers differ from population density. stations

Thank you!

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Codes and first draft to be published soon